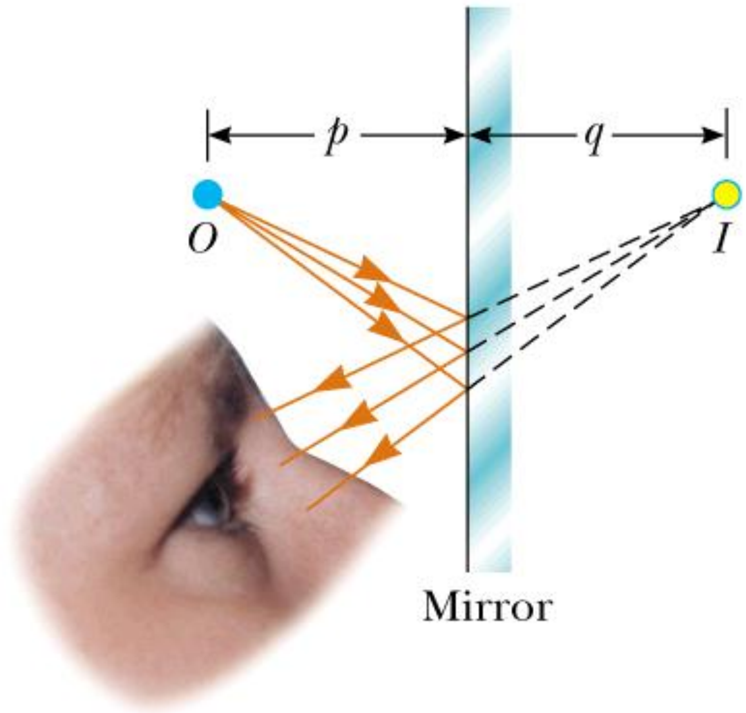
Geometric Optics

- Flat Mirrors
- Spherical Mirrors
- Images Formed by Refraction
- Thin Lenses
- Optical Instruments

Images - Terminology



p: Object Distance

q: Image Distance

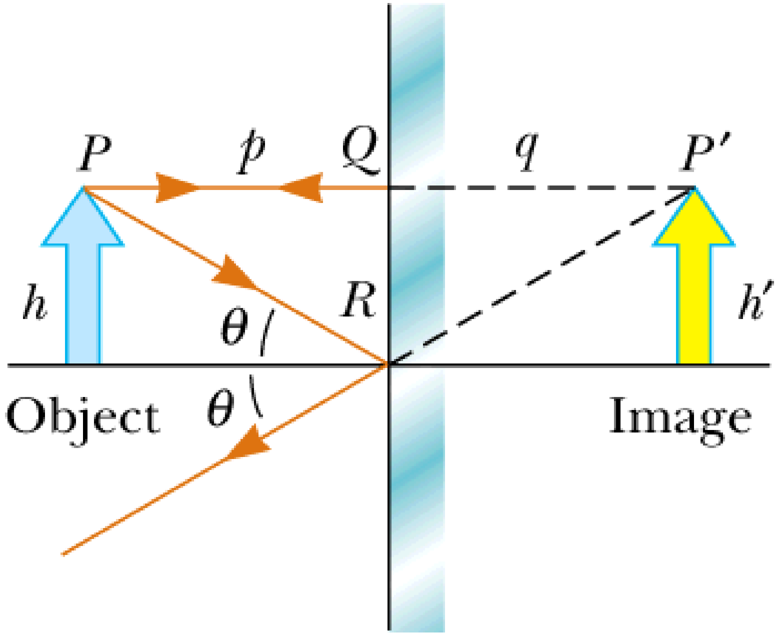
Real Images: When light rays pass through and diverge from the image point.

Virtual Images: When light rays do not pass through but appear to diverge from the image point.

Magnification

$$M \equiv \frac{\text{Image Height}}{\text{Object Height}} = \frac{h'}{h}$$

Images Formed by Flat Mirrors



$$p = q$$

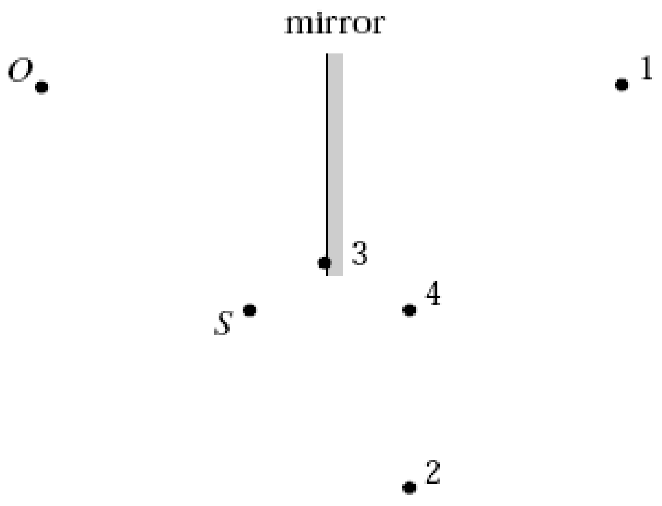
The image is virtual

For flat mirrors, $M = 1$

- The image distance is equal to the object distance.
- The image is unmagnified, virtual and upright.
- The image has front-back reversal.

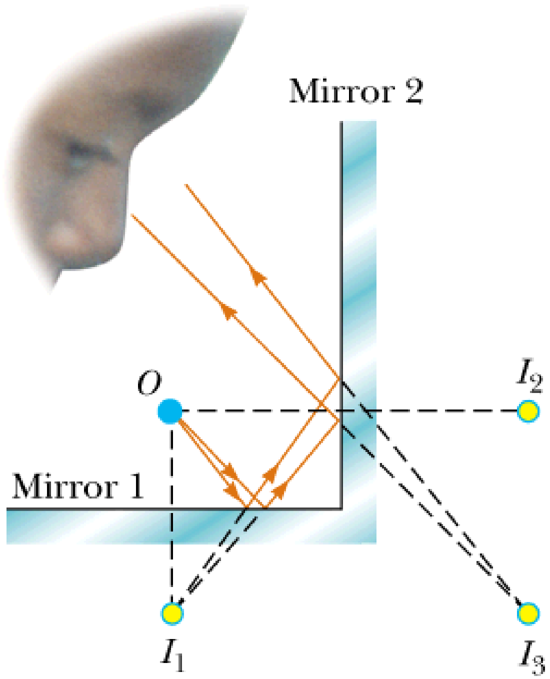
Concept Question

An observer *O*, facing a mirror, observes a light source *S*. Where does *O* perceive the mirror image of *S* to be located?



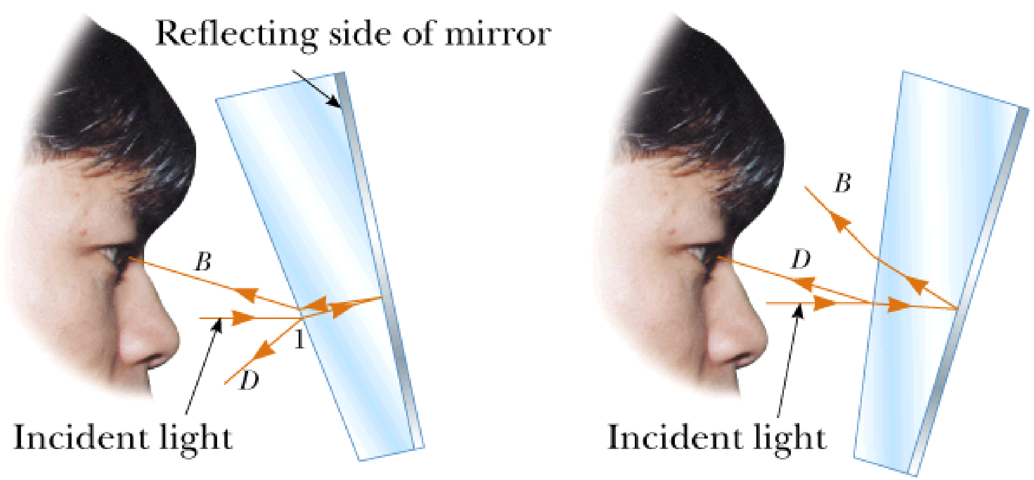
1. 1
2. 2
3. 3
4. 4
5. Some other location.
6. The image of *S* cannot be seen by *O* when *O* and *S* are located as shown.

Some Examples



Multiple Images
Formed by Two Mirrors

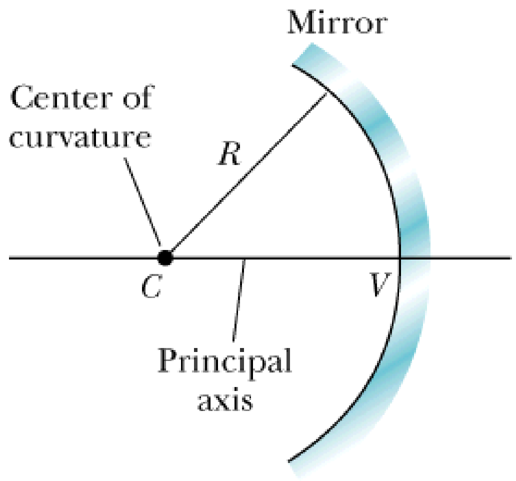
Rearview
Mirror



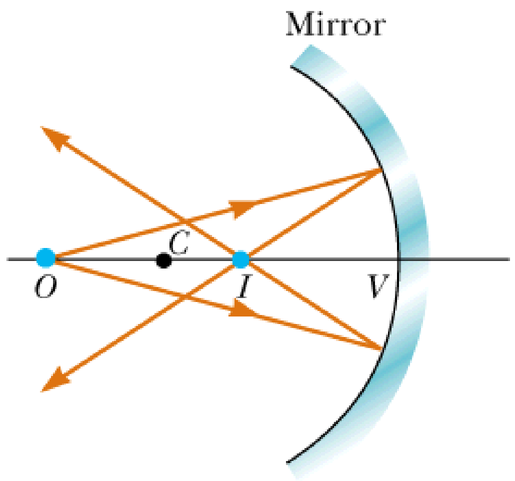
(a) Daytime setting

(b) Nighttime setting

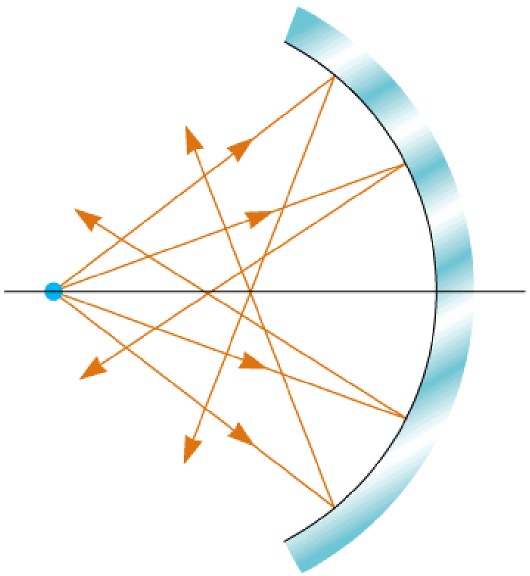
Concave Spherical Mirrors



Spherical Concave Mirror



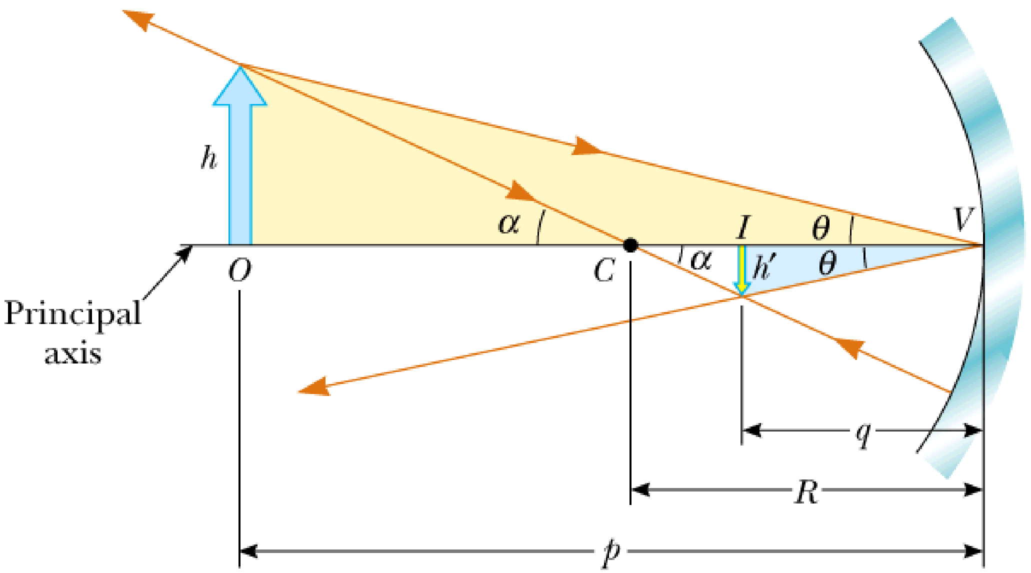
A real image is formed by a concave mirror

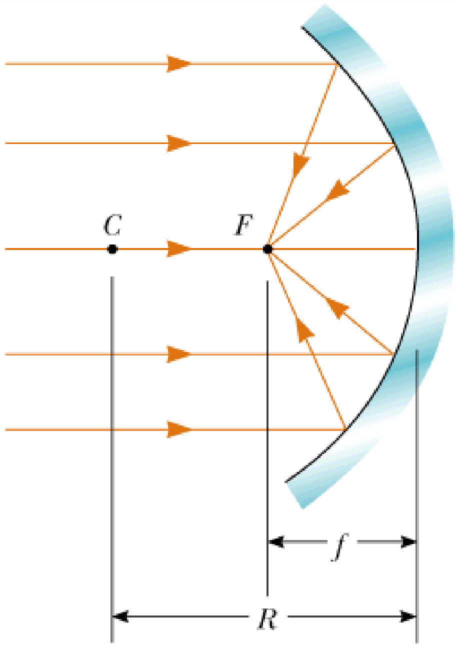


Spherical Aberration

Paraxial Approximation: Only consider rays making a small angle with the principal axis

Image Formation





$$\tan\theta = \frac{h}{p} = -\frac{h'}{q}$$

$$\tan\alpha = \frac{h}{p-R} = -\frac{h'}{R-q}$$

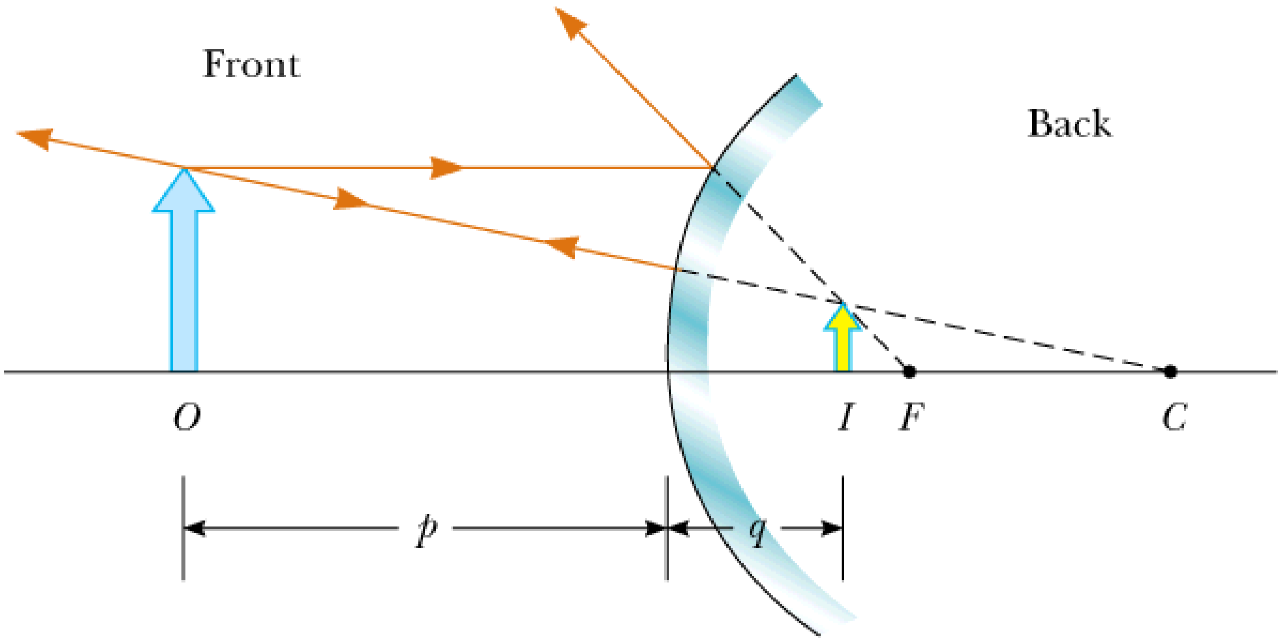
$$M = \frac{h'}{h} = -\frac{q}{p}$$

$$\frac{h'}{h} = -\frac{R-q}{p-R}$$

$$\frac{R-q}{p-R} = \frac{q}{p} \longrightarrow \frac{1}{p} + \frac{1}{q} = \frac{2}{R} \longrightarrow \frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

Focal Point $f = \dfrac{R}{2}$

Convex Spherical Mirrors

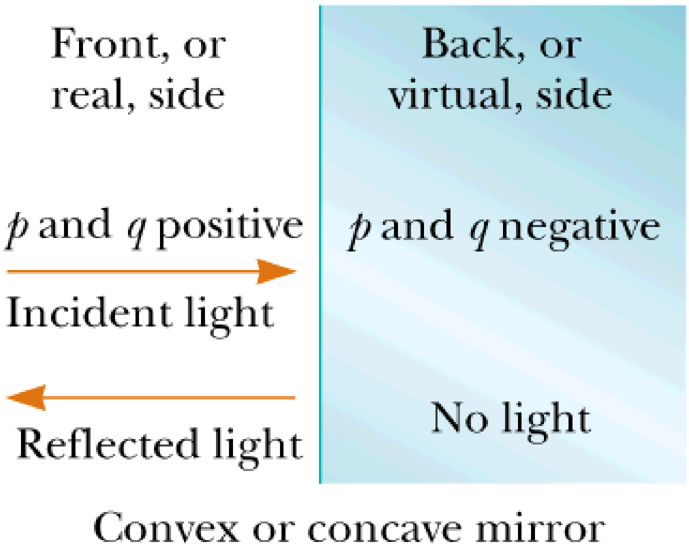


The image formed is upright and virtual

$$M = \frac{h'}{h} = -\frac{q}{p}$$

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

Sign Conventions for Mirrors



- p is **positive** if object is in **front** of mirror (real object).
- p is **negative** if object is in **back** of mirror (virtual object).

- q is **positive** if image is in **front** of mirror (real image).
- q is **negative** if image is in **back** of mirror (virtual image).

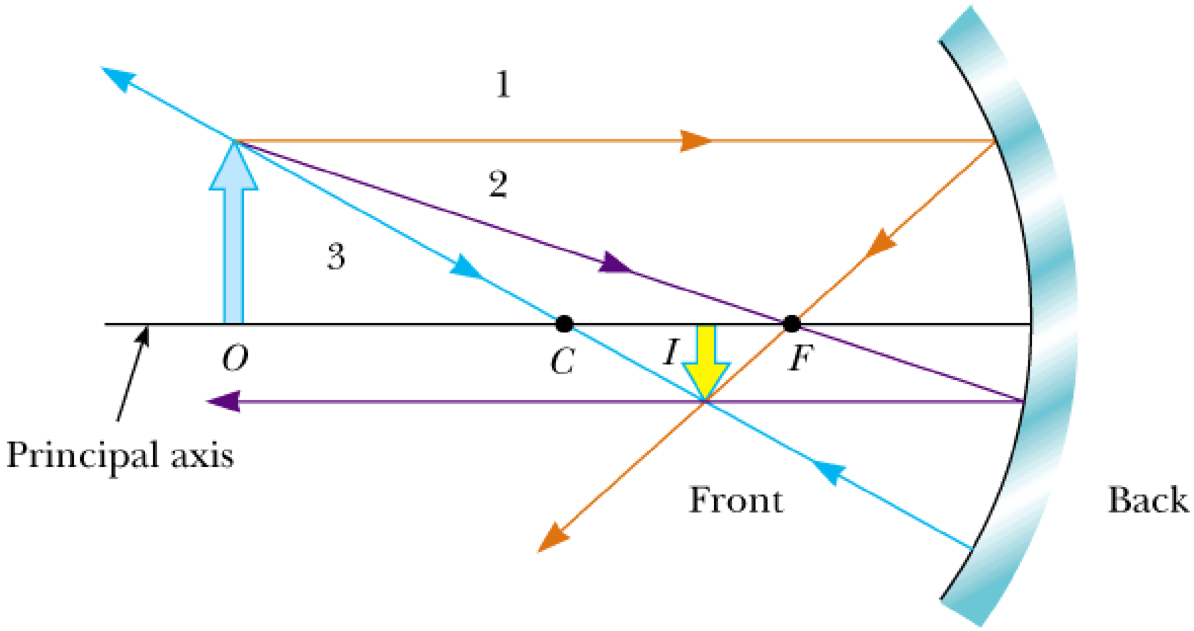
- Both f and R are **positive** if center of curvature is in **front** of mirror (concave mirror).
- Both f and R are **negative** if center of curvature is in **back** of mirror (convex mirror).

- If M is **positive**, image is **upright**.
- If M is **negative**, image is **inverted**.

Ray Diagrams For Mirrors

- **Ray 1** is drawn from the top of the object parallel to the principal axis and is reflected through the focal point F.
- **Ray 2** is drawn from the top of the object through the focal point and is reflected parallel to the principal axis.
- **Ray 3** is drawn from the top of the object through the center of curvature C and is reflected back on itself.

Concave Mirror ($p > R$)



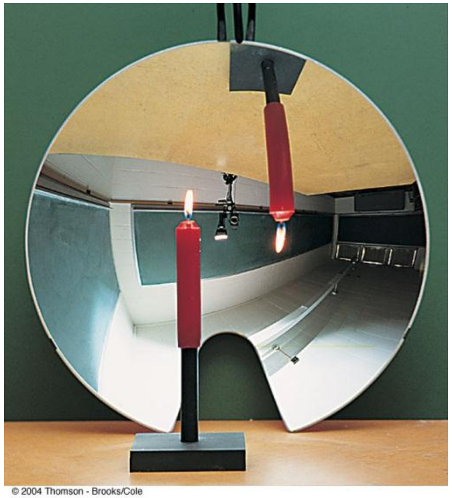
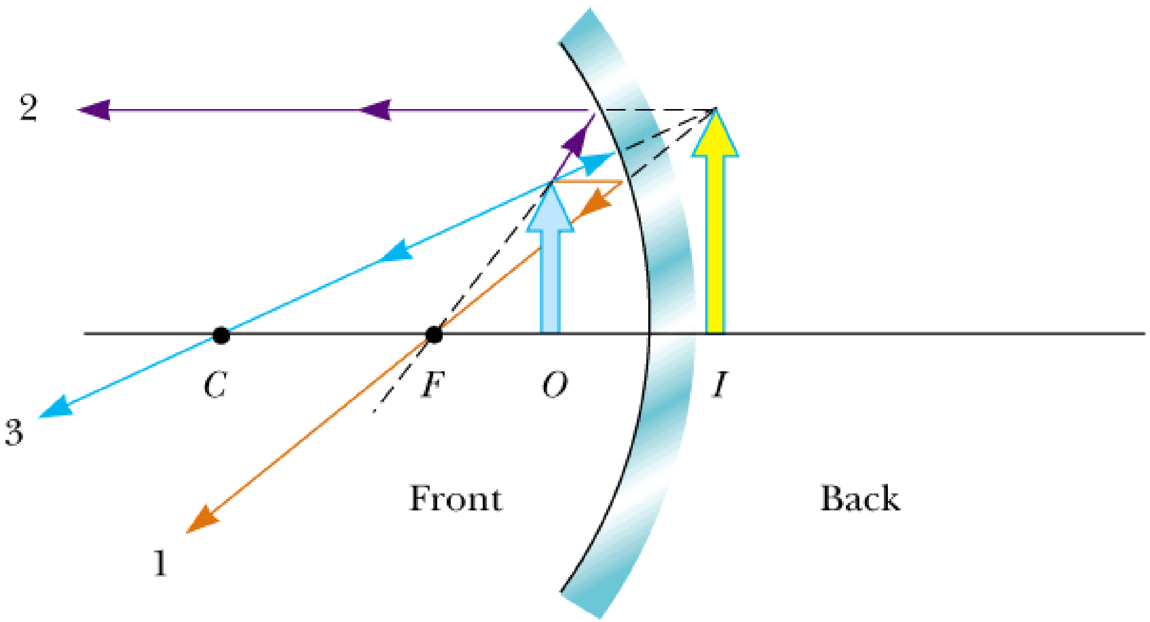


Image is real, inverted and smaller than the object

Concave Mirror ($p < f$)



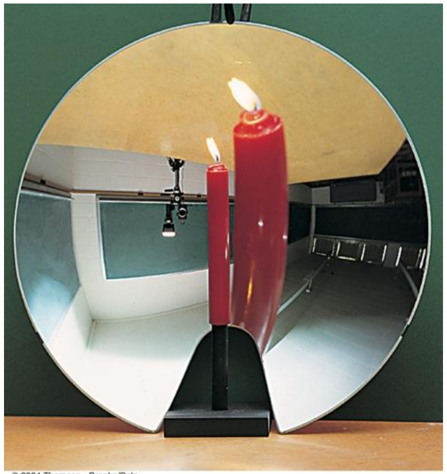


Image is virtual, upright and larger than the object

Convex Mirror

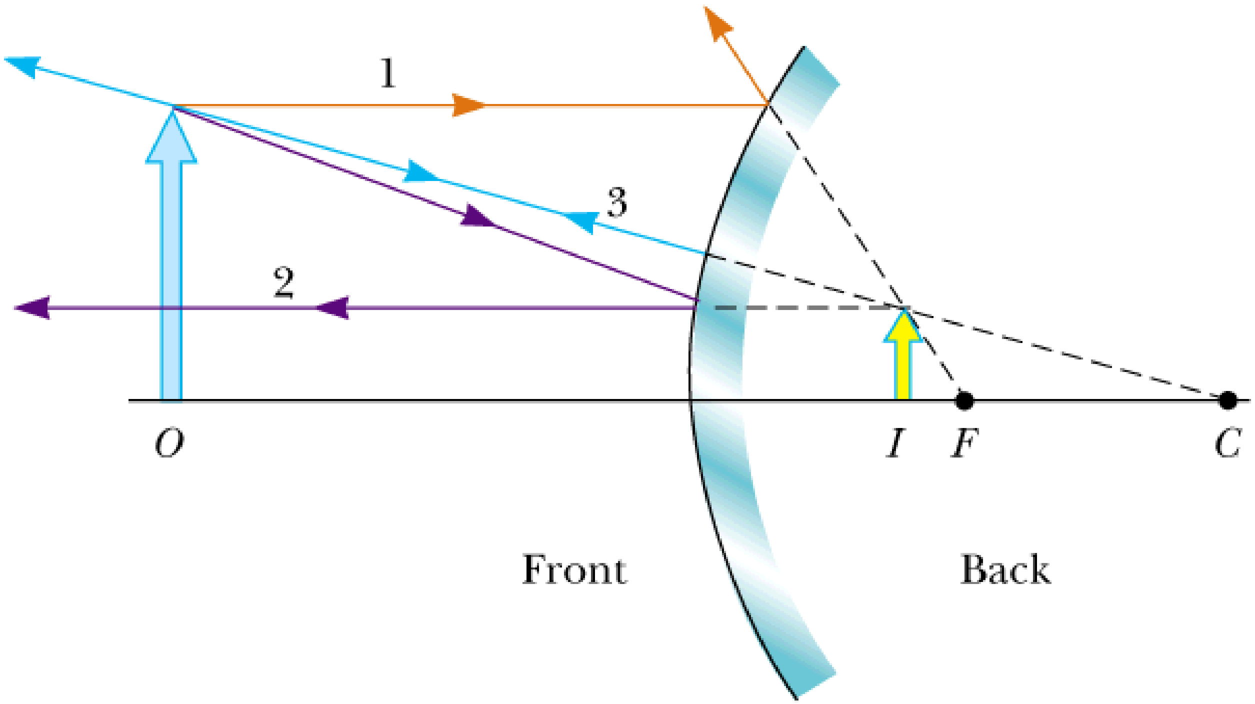
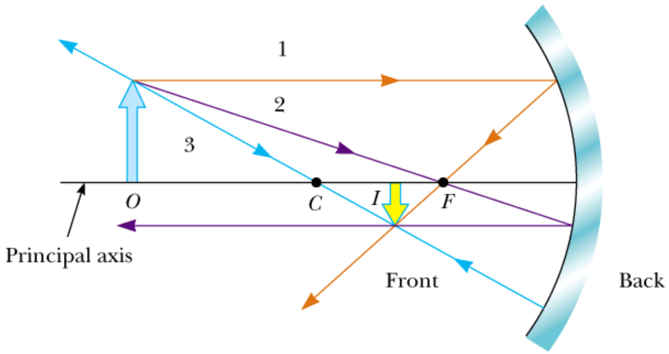


Image is virtual, upright and smaller than the object

Image From a Mirror

$f = +10$ cm ⟶ Concave Mirror

(a) p = 25 cm



$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$

$$\frac{1}{25}+\frac{1}{q}=\frac{1}{10} \longrightarrow q = 16.7cm$$

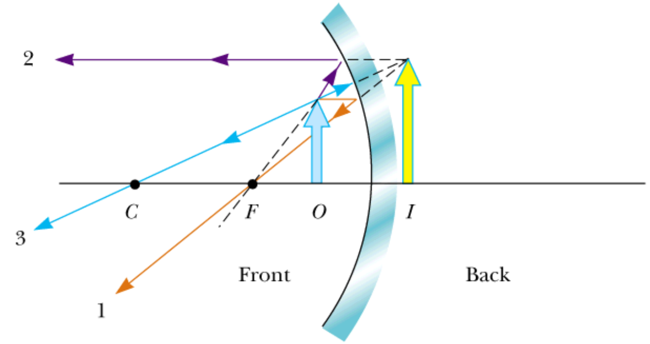
$$M = \frac{h'}{h} = -\frac{q}{p} = -0.668$$

(b) p = 10 cm

$$\frac{1}{10}+\frac{1}{q}=\frac{1}{10}$$

$$q = \infty$$

(c) p = 5 cm

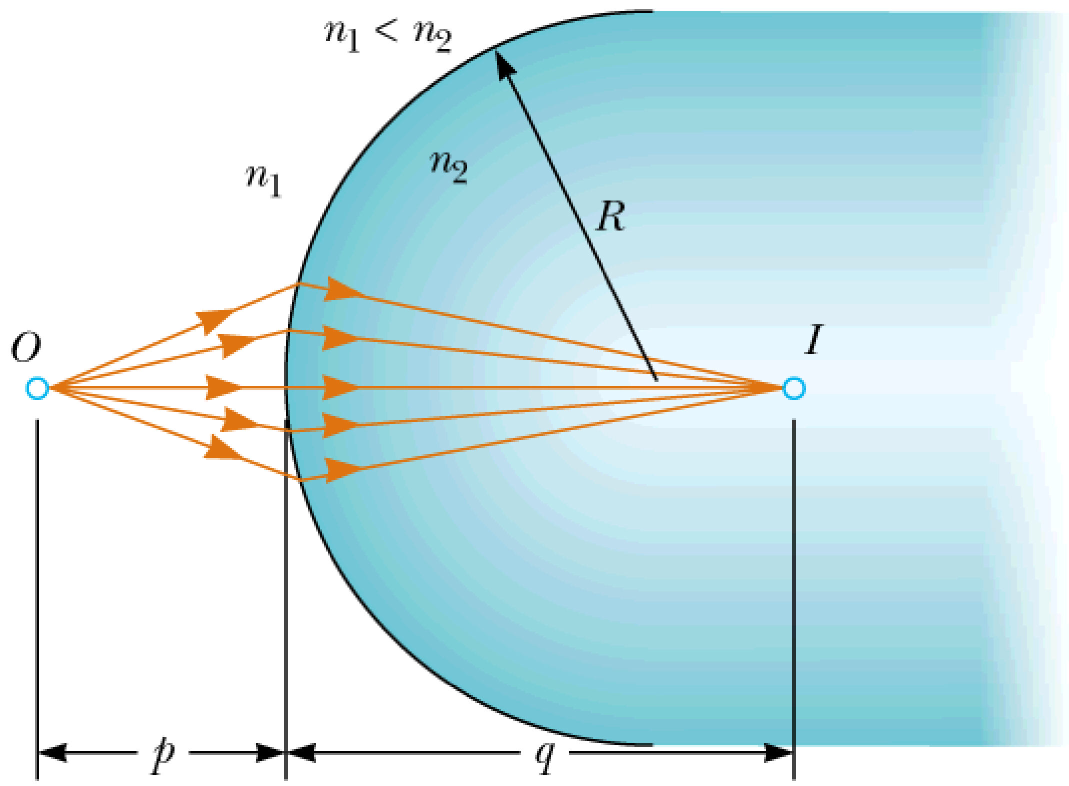


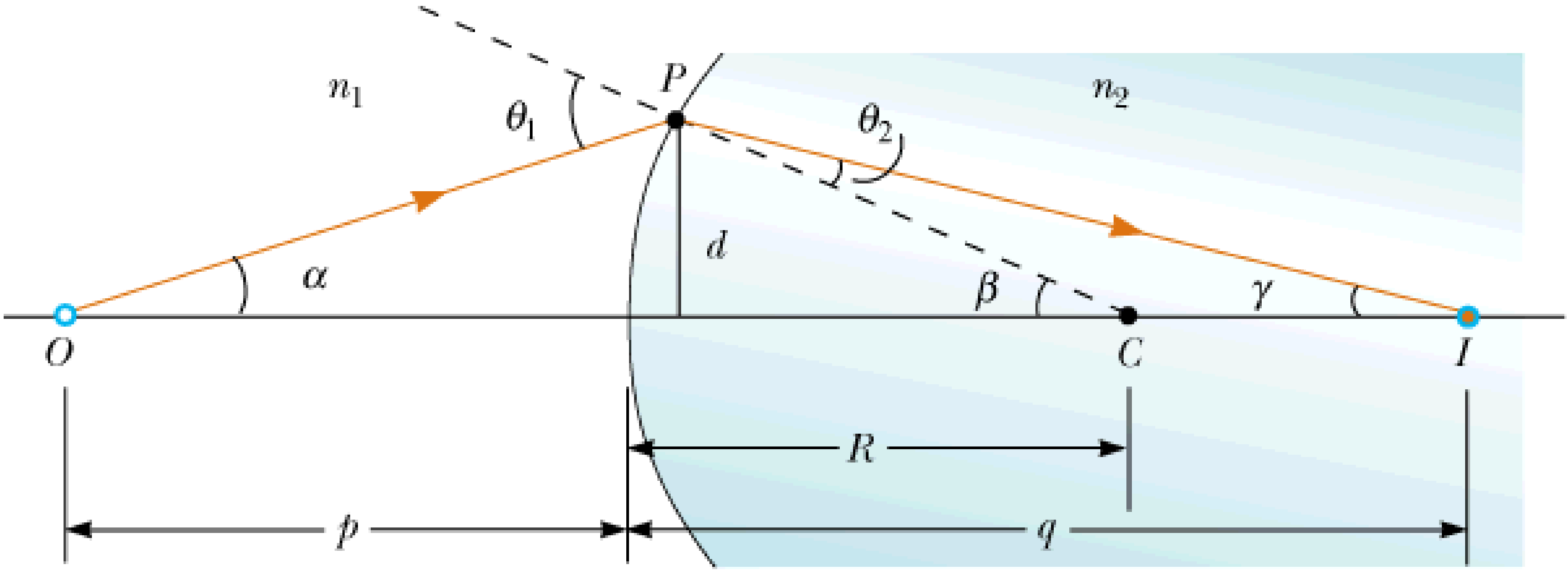
$$\frac{1}{5}+\frac{1}{q}=\frac{1}{10}$$

$$q = -10cm$$

$$M = \frac{h'}{h} = -\frac{q}{p} = 2$$

Images Formed By Refraction





$$n_1 Sin\theta_1 = n_2 Sin\theta_2 \longrightarrow n_1\alpha + n_2\gamma = (n_2 - n_1)\beta \longrightarrow n_1\frac{d}{p} + n_2\frac{d}{q} = (n_2 - n_1)\frac{d}{R}$$

$$n_1\theta_1 \approx n_2\theta_2 \qquad \tan\alpha \approx \alpha \approx \frac{d}{p}$$

$$\theta_1 = \alpha + \beta \qquad \tan\beta \approx \beta \approx \frac{d}{R}$$

$$\frac{n_1}{p} + \frac{n_2}{q} = \frac{(n_2 - n_1)}{R}$$

$$\beta = \theta_2 + \gamma \qquad \tan\gamma \approx \gamma \approx \frac{d}{q}$$

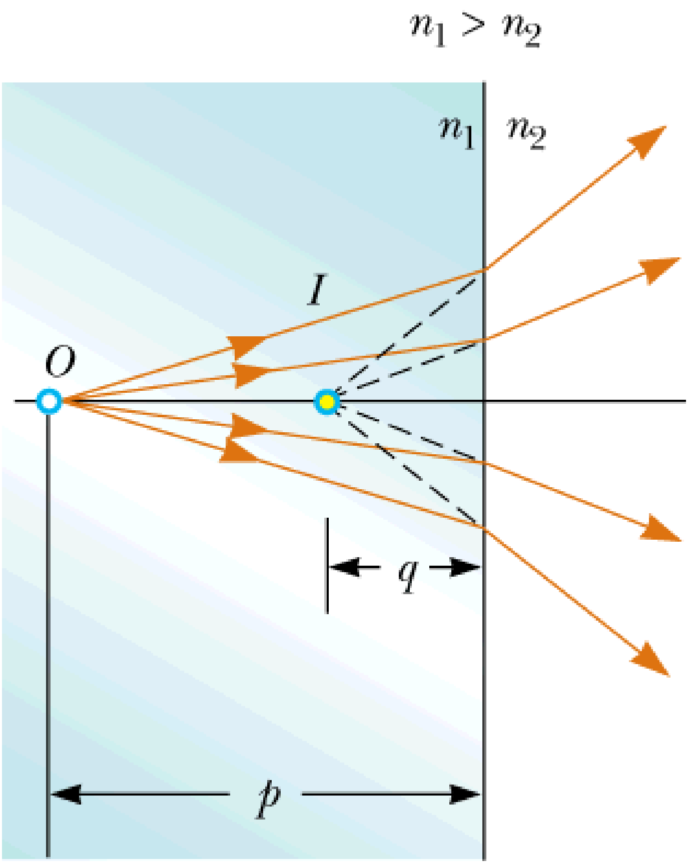
Sign Conventions for Refracting Surfaces

- p is **positive** if object is in **front** of surface (real object).
- p is **negative** if object is in **back** of surface (virtual object).

- q is **positive** if image is in **back** of surface (real image).
- q is **negative** if image is in **front** of surface (virtual image).

- R is **positive** if center of curvature is in **back** of convex surface.
- R is **negative** if center of curvature is in **front** of concave surface.

Flat Refracting Surface



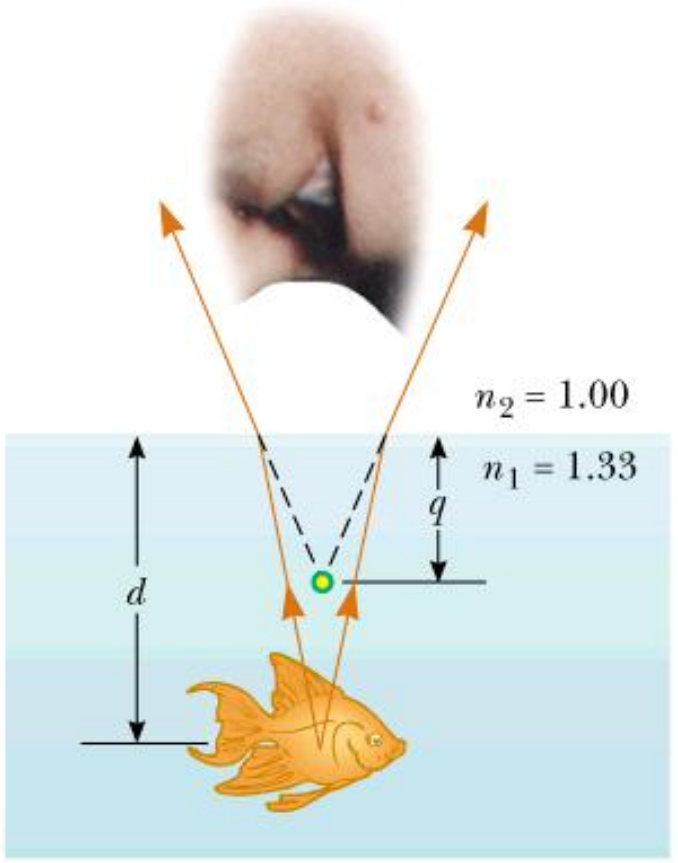
$$R = \infty$$

$$\frac{n_1}{p} + \frac{n_2}{q} = 0$$

$$q = -\frac{n_2}{n_1} p$$

The image is on the same side of the surface as the object.

Apparent Depth



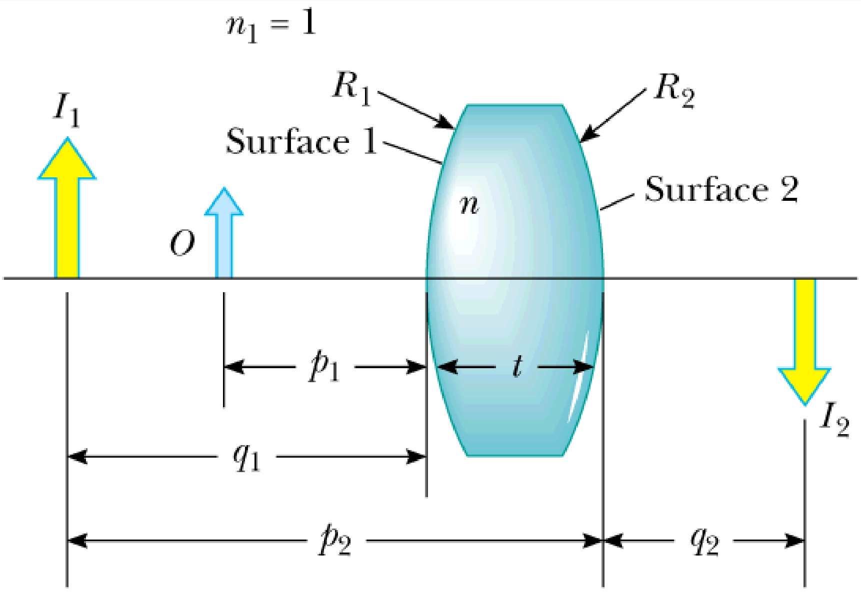
$$p = d$$

$$q = -\frac{n_2}{n_1} p$$

$$q = -\frac{1}{1.33} d = -0.752d$$

The image is virtual

Thin Lenses



The image formed by the first surface acts as the object for the second surface

$$\frac{1}{p_1} + \frac{n}{q_1} = \frac{(n-1)}{R_1}$$

where, $q_1 < 0$

$$\frac{n}{p_2} + \frac{1}{q_2} = \frac{(1-n)}{R_2}$$

$$p_2 = -q_1 + t \approx -q_1$$

$$-\frac{n}{q_1} + \frac{1}{q_2} = \frac{(1-n)}{R_2}$$

$$\frac{1}{p_1} + \frac{1}{q_2} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{p} + \frac{1}{q} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

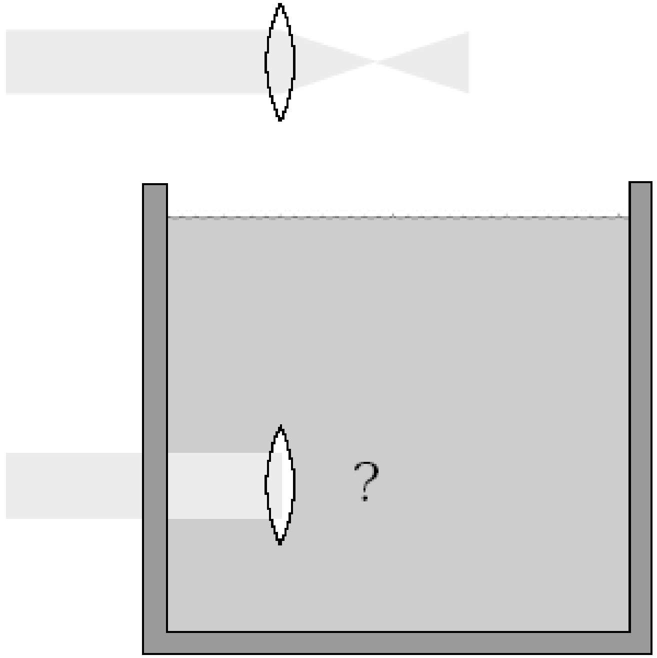
Lens Makers' Equation

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

$$M = \frac{h'}{h} = -\frac{q}{p}$$

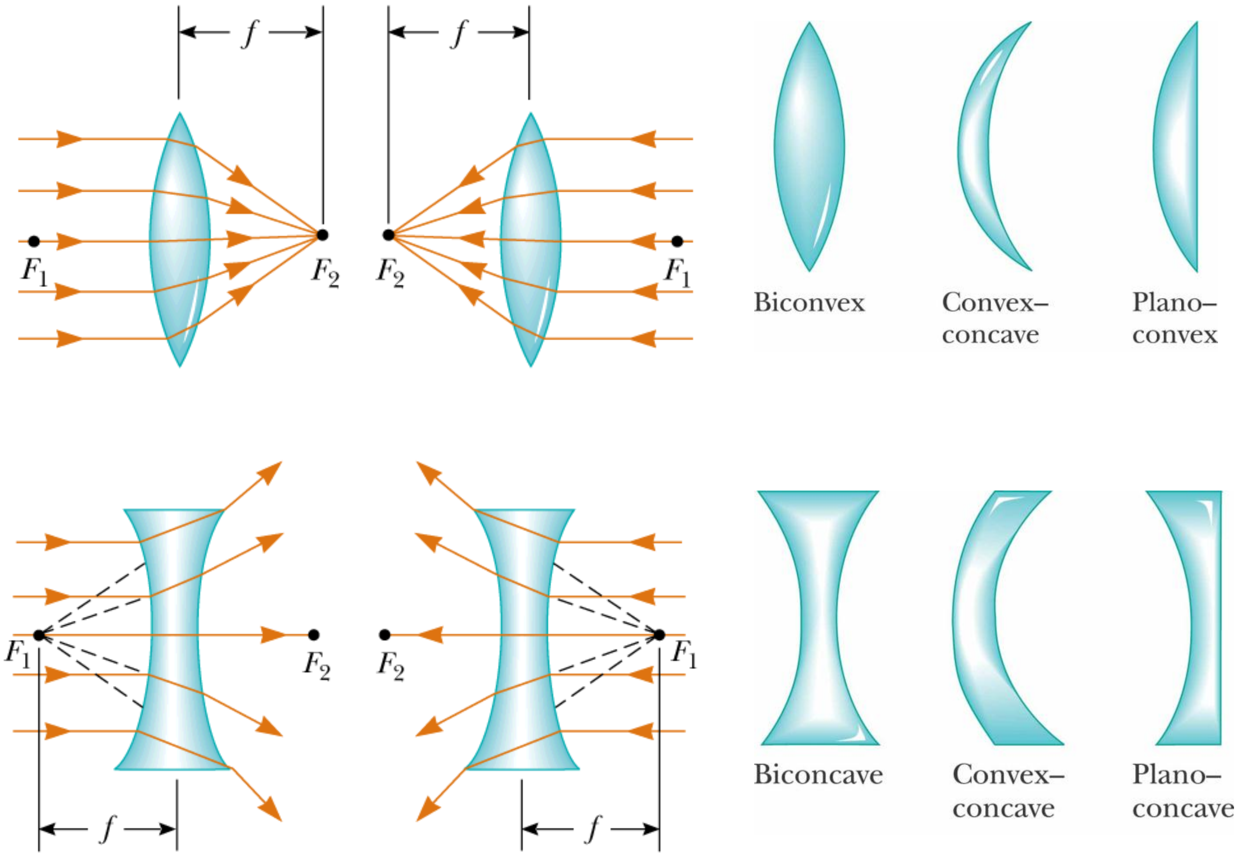
Concept Question

A parallel beam of light is sent through an aquarium. If a convex glass lens is held in the water, it focuses the beam



1. closer to the lens than
2. at the same position as
3. farther from the lens than outside the water.

Lens Types



Converging Lenses

f_1: object focal point
f_2: image focal point

Diverging Lenses

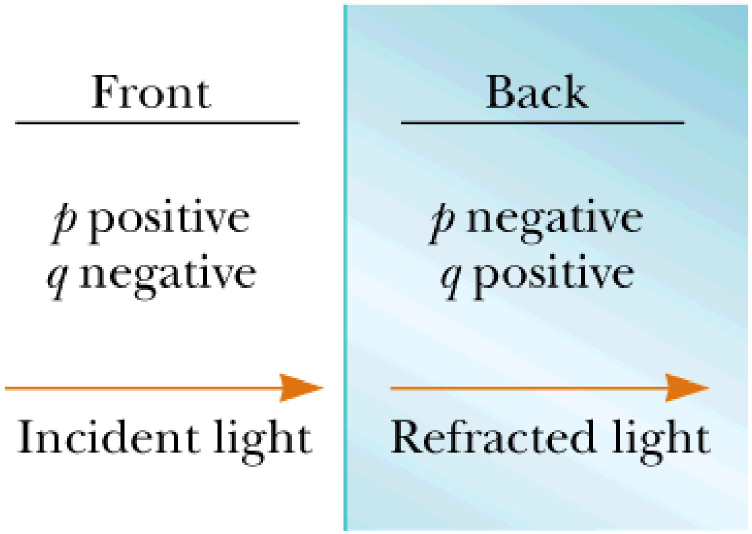
Sign Conventions for Thin Lenses

- p is **positive** if object is in **front** of lens (real object).
- p is **negative** if object is in **back** of lens (virtual object).

- q is **positive** if image is in **back** of lens (real image).
- q is **negative** if image is in **front** of lens (virtual image).

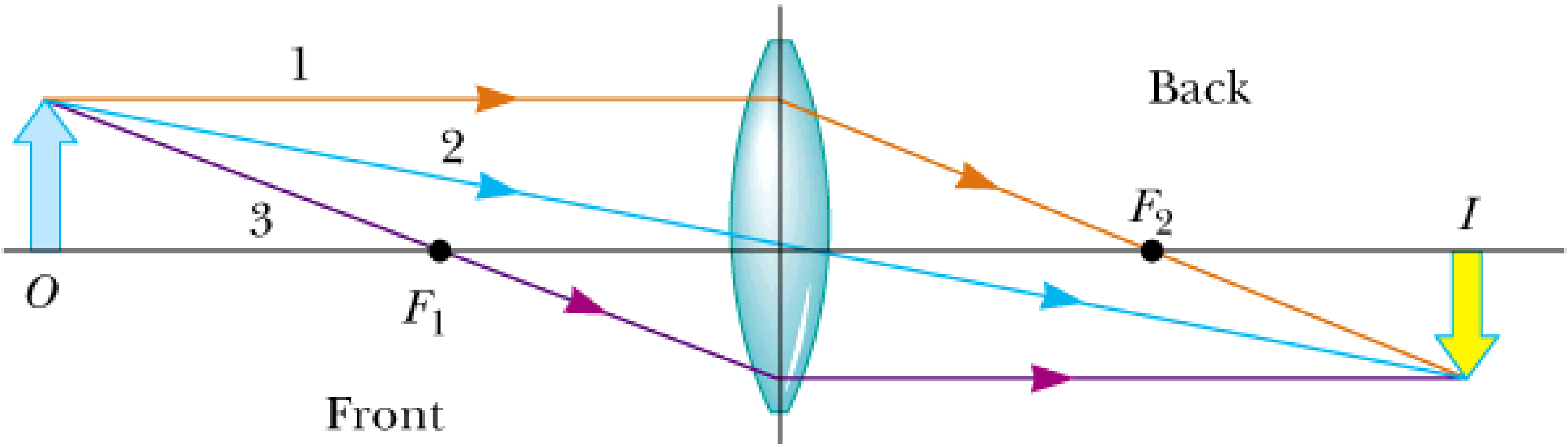
- R_1 and R_2 are **positive** if center of curvature is in **back** of lens.
- R_1 and R_2 are **negative** if center of curvature is in **front** of lens.

- f is **positive** if the lens is **converging**.
- f is **negative** if the lens is **diverging**.

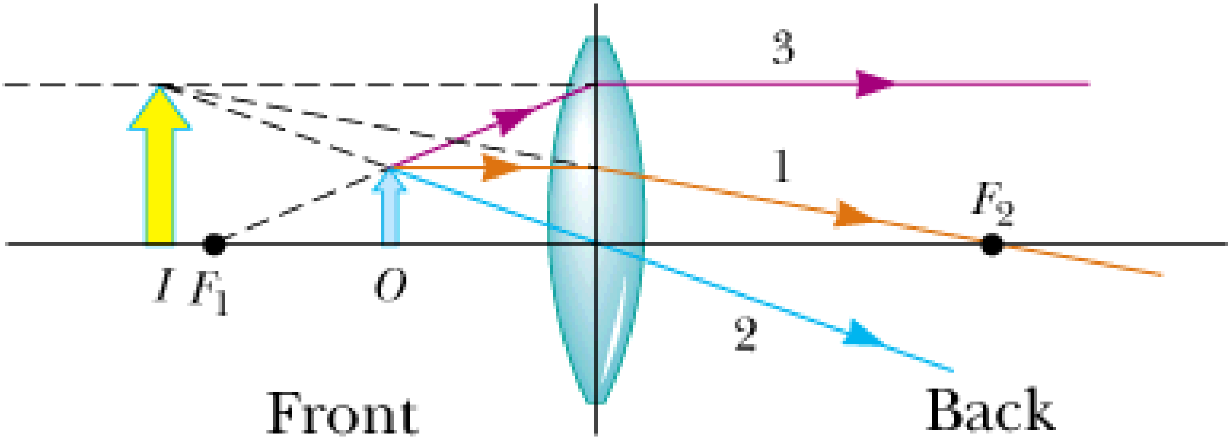


Ray Diagrams for a Converging Lens

- **Ray 1** is drawn parallel to the principal axis. After being refracted, this ray passes through the focal point on the back side of the lens.
- **Ray 2** is drawn through the center of the lens and continues in a straight line.
- **Ray 3** is drawn through the focal point on the front side of the lens (or as if coming from the focal point if $p < f$) and emerges from the lens parallel to the principal axis.



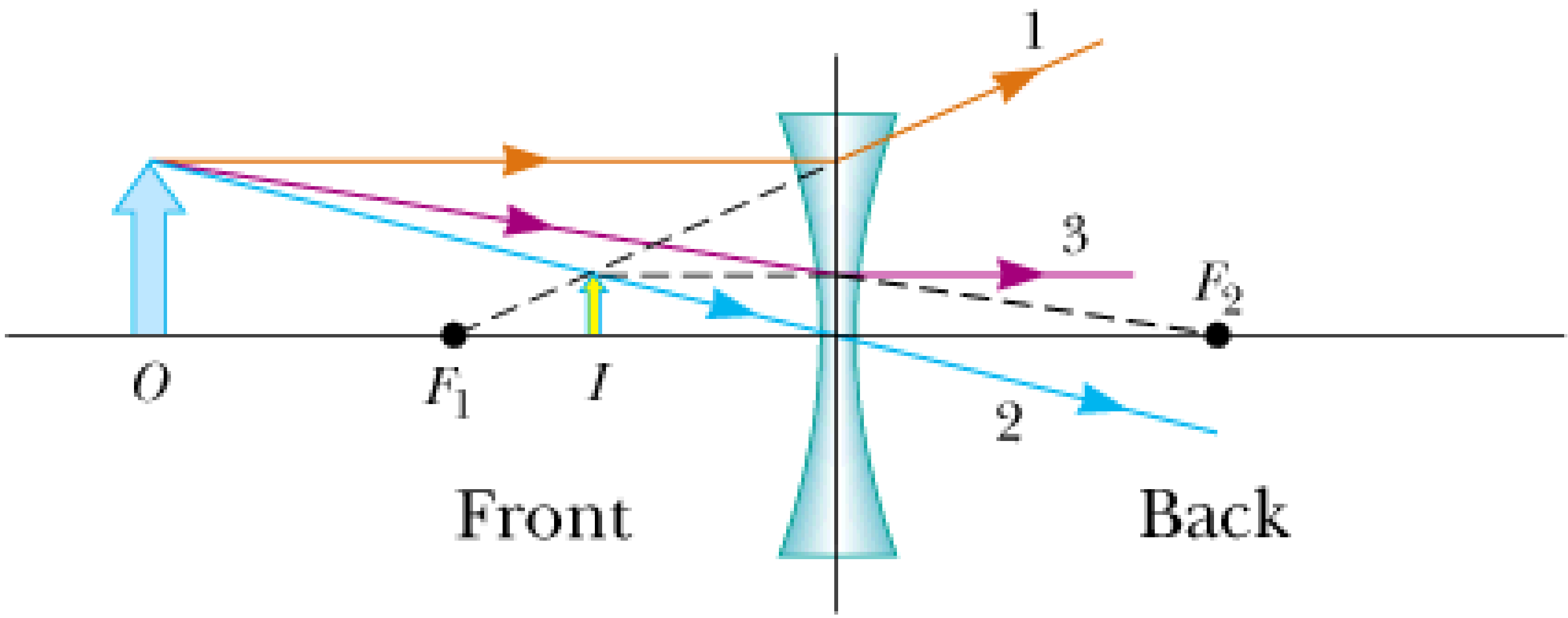
The image is real and inverted



The image is virtual and upright

Ray Diagrams for a Diverging Lens

- **Ray 1** is drawn parallel to the principal axis. After being refracted, this ray emerges such that it appears to have passed through the focal point on the front side of the lens.
- **Ray 2** is drawn through the center of the lens and continues in a straight line.
- **Ray 3** is drawn toward the focal point on the back side of the lens and emerges from the lens parallel to the principal axis.



The image is virtual and upright

Examples

A diverging lens with f = -20 cm
h = 2 cm, p = 30 cm

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$

$$\frac{1}{30}+\frac{1}{q}=\frac{1}{-20}$$

$$q=-12cm$$

$$M=\frac{h'}{h}=-\frac{q}{p}$$

$$M=\frac{h'}{2}=-\frac{-12}{30}=0.4$$

$$h'=0.8cm$$

The image is virtual and upright

A converging lens with f = 10 cm

(a) p = 30 cm

$$\frac{1}{30}+\frac{1}{q}=\frac{1}{10}$$

$$q=15cm$$

$$M=-\frac{q}{p}=-\frac{15}{30}=-0.5$$

The image is real and inverted

(b) p = 10 cm

$$q=\infty$$

The image is at infinity

(c) p = 5 cm

$$\frac{1}{5}+\frac{1}{q}=\frac{1}{10}$$

$$q=-10cm$$

$$M=-\frac{q}{p}=-\frac{-10}{5}=2$$

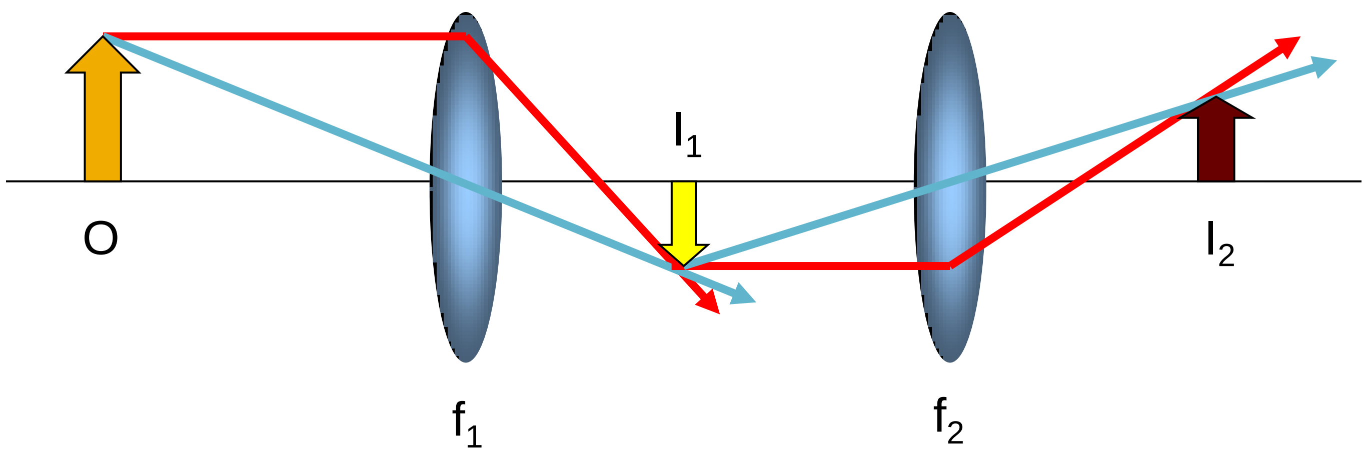
The image is virtual and upright

Java Applet for Lens and Mirrors

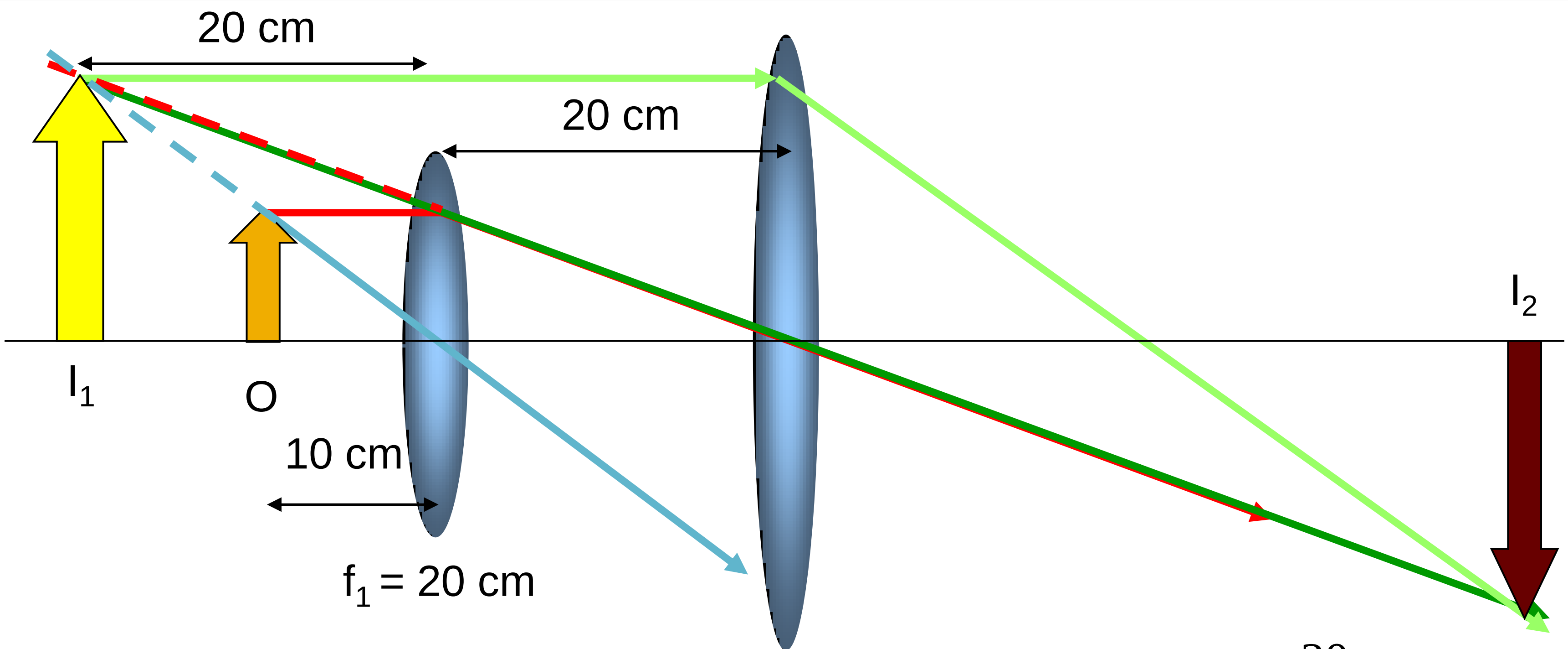
- http://www.phy.ntnu.edu.tw/java/index.html

Combination of Thin Lenses

- First find the image created by the first lens as if the second lens is not present.
- Then draw the ray diagram for the second lens with the image from the first lens as the object.
- The second image formed is the final image of the system.



Example



f_2 = 20 cm

$$\frac{1}{p_1}+\frac{1}{q_1}=\frac{1}{f_1} \qquad \frac{1}{p_2}+\frac{1}{q_2}=\frac{1}{f_2}$$

$$\frac{1}{10}+\frac{1}{q_1}=\frac{1}{20} \qquad \frac{1}{40}+\frac{1}{q_2}=\frac{1}{20}$$

$$q_1=-20cm \qquad q_2=40cm$$

$$M_1=-\frac{q_1}{p_1}=-\frac{-20}{10}=2$$

$$M_2=-\frac{q_2}{p_2}=-\frac{40}{40}=-1$$

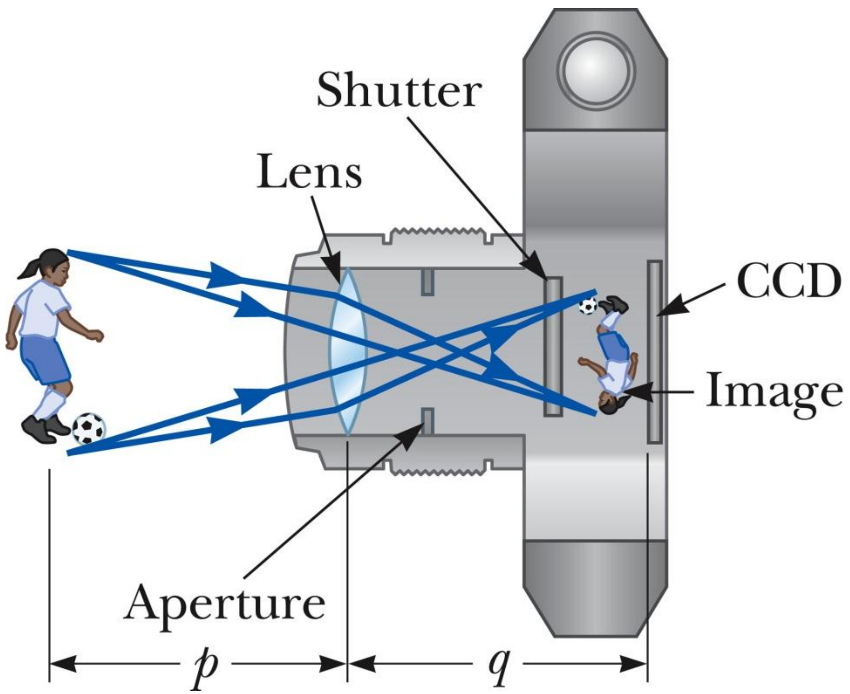
$$M=M_1M_2=(2)(-1)=-2$$

The Camera

- A lens is used to form an image of an object on the film (or detector array).
- The amount of light entering the camera is controlled by the aperture.
- The exposure is controlled by the shutter speed.

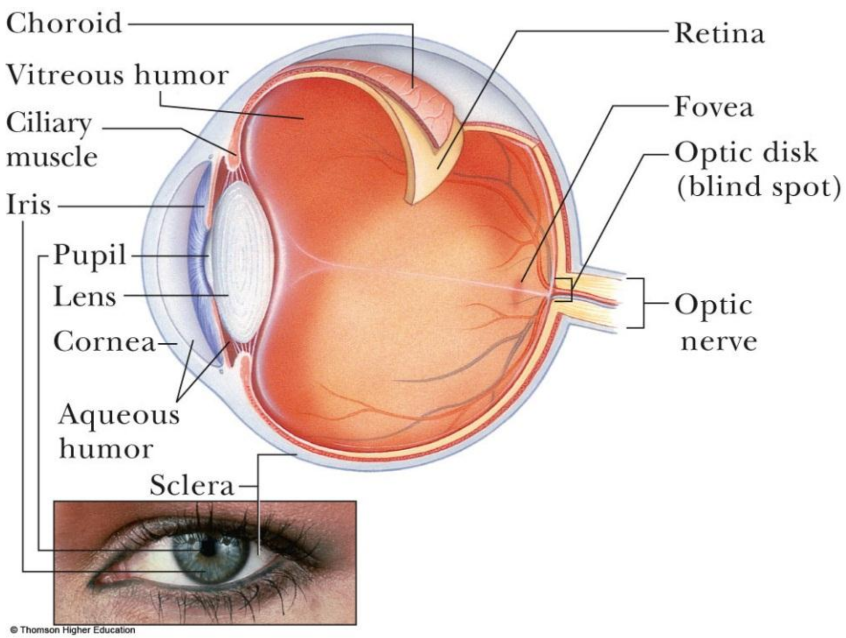
$$I \propto D^2 / f^2$$

$$f - number = \frac{f}{\#} = \frac{f}{D}$$



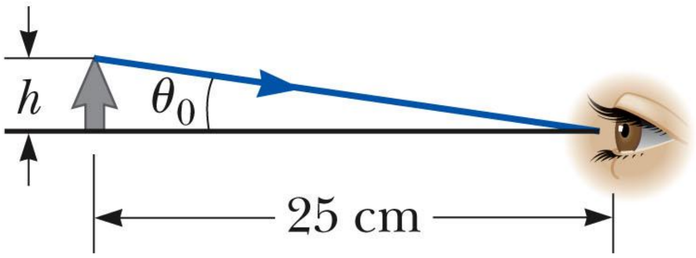
The Eye

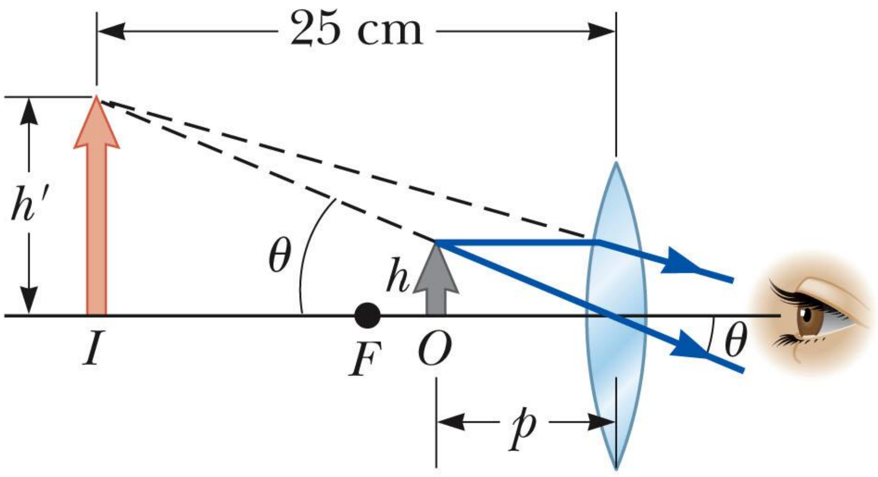
- Light is refracted by the cornea (which includes an aqueous humor and lens) and its intensity is regulated by the iris.
- Light ideally focuses on the retina which has a set of receptors called the rods and cones.
- The receptors send optical information to the brain via the optical nerve.
- Focusing is done by changing the shape (curvature) of the lens.
- The closest point of focus is the near point (~ 25 cm).



The Simple Magnifier

Use a lens near the eye to make an object seem larger (occupy a larger angle at the eye).





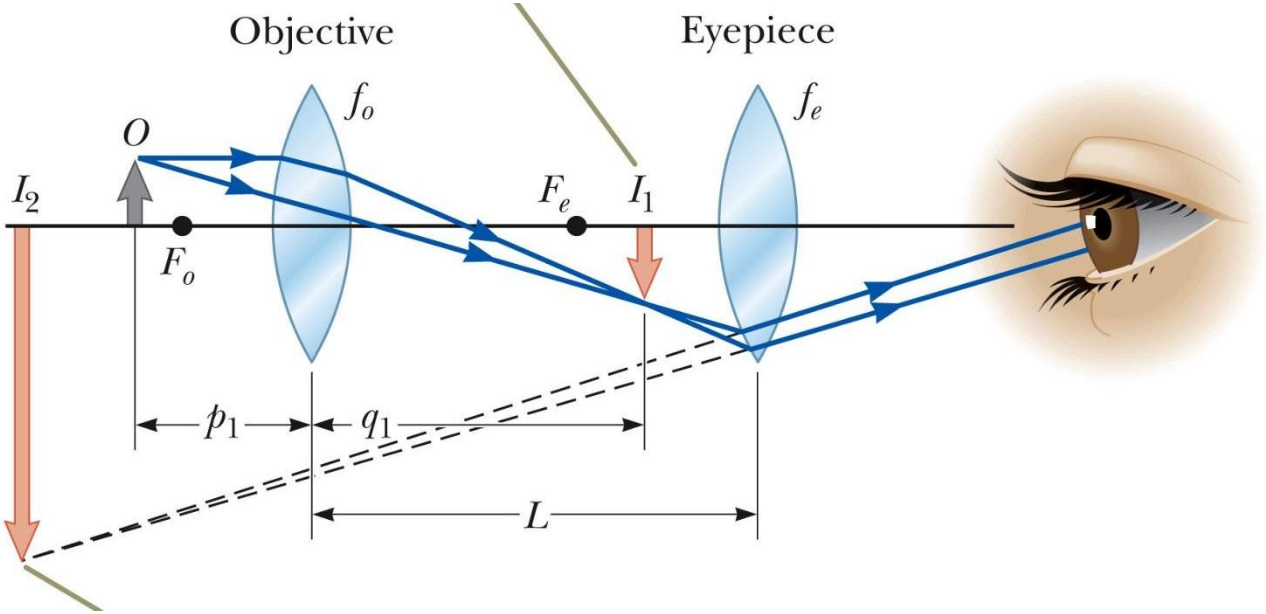
$$m_\theta = \frac{\theta'}{\theta}$$

$$m_\theta \approx \frac{25cm}{f}$$

$$m_{\theta.\max} \approx \frac{25cm}{f} + 1$$

Compound Microscope

Use a lens combination to make small objects near the objective seem more visible.



$$m = -\frac{q}{p} \approx -\frac{L}{f_o}$$

$$M = m m_\theta = -\frac{L}{f_o}\frac{25cm}{f_e}$$

For Next Class

- Midterm 3 Review on Tuesday
- Midterm 3 on Wednesday
- Reading Assignment for Thursday
  - Chapter 37: Interference of Light Waves
- WebAssign: Assignment 14 due Tuesday, 11 pm